The Development of Adaptive Reasoning in the Mathematics Classroom

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Abstract: Each strand of mathematical proficiency requires the development of sometimes overlapping skills. Instead of debating which strand demands the most attention, mathematics educators have been discussing ways in which to adequately develop every strand. Adaptive reasoning is a set of skills that can be difficult to develop and may require teachers to adjust their established teaching practices. If educators have access to successful researched-based instructional strategies, they will be better prepared to develop these skills in their students. There is widespread agreement that students who are unable to adequately reason will have limited mathematical proficiency, and there are several research-based strategies educators can use to develop adaptive reasoning skills in young students of mathematics.

Introduction

Imagine a mathematics class full of sixth grade students at the beginning of the school year. They spent the previous year extending their knowledge of fractions and learned how to fluently add and subtract simple fractions and mixed numbers. Additionally, they were taught how to multiply and divide fractions with whole numbers and other fractions. The class is presented with a problem designed to activate their prior knowledge and assess how well they remember the concept of dividing a whole number by a fraction. On the board, the teacher writes, "Timmy has 6 sub sandwiches that are each divided into sections that are 1/4 the length of the sub sandwich. How many total sections does Timmy have?" The students are asked to find a solution and write an equation that represents the situation.

The teacher notices multiple students looking frustrated and looks at the work of one student. She has written "6 ÷ 1/4 =?" He asks what she is thinking, and she responds: "I know you have to divide, but I don't remember how to divide by a fraction. I think there was something where you flip the fraction, but I don't remember." The teacher encourages her to try to think of other ways to find the solution without needing to remember that procedure. He finds several other students are stuck for the same reason. It seems they have forgotten the procedure for dividing a whole number by a fraction, and they are unable to adapt their thinking to find an alternative path to the solution.

Another student has forgotten the procedure as well, but he has used the strategy of modeling. He draws out six sub sandwiches and draws lines as if he is cutting them all into fourths. Then, he counts each section to come up with the correct answer of 24. Other students find the solution in a similar manner by reasoning that if the sub was cut into sections that are 1/4 of a sub long, then there are four sections per sub. Simply multiplying that number by six also yields the correct answer of 24. This reasoning can be used to justify the procedure that the first group of students had forgotten.

The final set of students has the solution written as "6 ÷ 1/4 = 24". When asked how they found the solution, they state that they remember when dividing a number by a fraction, the fraction needs to be flipped and the division needs to be changed to multiplication. This is known as the "invert and multiply" rule. This rule is simply a trick, or a shortcut used to quickly calculate solutions to problems involving division by a fraction. When asked why the student used that rule, they reason that it was the way they were taught. When asked why that rule works, the students are unable to justify the reasoning behind it. They simply state, "that's the way we were taught."

This scenario is an example of how students may think when faced with an exercise or problem in mathematics. The teacher wants his students to understand concepts, know when and how to use procedures, and understand that math is important and worthwhile. He also understands that his students need to be able to reason through the problem-solving process and be able to justify their solutions, however, he is unsure how to develop these skills in his students. In this paper, adaptive reasoning will be defined and its importance in mathematical proficiency will be established. Several research-based strategies to promote the development of adaptive reasoning in the mathematics classroom will be discussed. Using these strategies with young students of mathematics can enable them to develop skills necessary for mathematical proficiency.

What is Adaptive Reasoning?

Adaptive reasoning refers to the capacity to think logically about the relationships among concepts and situations (National Research Council [NRC], 2001). Students who have developed this skill can use prior knowledge and a variety of solution methods to reach a conclusion about problems, including ones which are unfamiliar. These students can also justify their procedures and conclusions using formal and informal reasoning. Students display procedural fluency when they know mathematical procedures and can use them appropriately, accurately, and efficiently (NRC, 2001). Procedural fluency is an important strand of mathematical proficiency, but students are often taught to rely on certain memorized facts and procedures without having to justify their reasoning behind the use of those procedures. Even now, knowing this reliance on procedural knowledge is detrimental to a student's deeper understanding of mathematics, teachers still focus on this strand.

The NRC (2001) claims that "adaptive reasoning is the glue that holds everything together" (p. 129). The "everything" they are referring to are the five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Knowing what those strands are and why they are important for mathematics education is not as important as being able to effectively teach those skills. Mathematics education that relies on students to follow a certain procedure or set of rules without logical thought or the ability to explain and justify a solution will result in students with a lack of overall mathematical understanding. An example provided by the NRC (2001) showed that when 13-year old students were asked to estimate 12/13+7/8 and given the choices of 1, 2, 19, and 21, 55% chose either 19 or 21. If a student does not remember the procedure for adding fractions, reasoning skills can be used to conclude that since both fractions are almost one whole, the actual answer should be close to two.

There is clearly a need for the development of adaptive reasoning in the mathematics classroom, and it is important for current and future educators to be equipped with specific instructional strategies to accomplish this. Battista (2017) claims:

> Students who achieve genuine understanding and sense making of mathematics are likely to stay engaged in learning it. Students who fail to understand and make sense of mathematical ideas and instead resort to rote learning will eventually experience continued failure and withdraw from mathematics learning. (p. 1)

Students can develop reasoning and sense making through the use of mathematics instruction based on three principles: (1) Students must construct these ideas for themselves as they try to make sense of situations; (2) Mathematics teaching must carefully guide and support students as they attempt to construct personally meaningful mathematical ideas in the context of problem solving, inquiry, and student discussion of multiple problem-solving strategies; and (3) Instruction must be derived from research-based descriptions of how students develop reasoning about particular mathematical topics (Battista, 2017).

What Can Teachers Do?

A review of research discovered that there are specific instructional strategies teachers can use to promote the development of adaptive reasoning in the mathematics classroom. Some of these strategies are commonly taught to preservice teachers through pedagogy instruction and training. These strategies are typically ongoing and may already be included in a teacher's instructional design. Other strategies include teaching models, such as the Creative Problem-Solving Model and the Problem-Based Learning Model. These models occur over longer periods of time, where individual skills that are necessary for students to succeed may need to be developed ahead of time.

Mathematics Interventions

Pulles and Burns (2022) examined mathematics interventions and how these interventions incorporated all five strands of mathematical proficiency as stated by the National Research Council. These interventions were defined as "instructional practices and activities designed to enhance the mathematics achievement of students" (Gersten et al., 2009, p. 1205). After reviewing 13 meta-analyses of studies of mathematics interventions that included students in grades kindergarten through eighth grade, eight interventions were found which had both a positive statistical effect and incorporated adaptive reasoning. These strategies include cognitive strategy instruction, concrete-representational-abstract, feedback, peer-assisted learning strategies, schema-based instruction, self-monitoring, self-regulated learning, and think-alouds (Pulles & Burns, 2022).

Cognitive strategy instruction combines instruction in cognitive and metacognitive strategies and processes with the purpose of teaching students how to think and behave like proficient problem solvers and strategic learners (Montague & Dietz, 2009). Using Montague's (1992) problem-solving model, students use cognitive strategies in seven steps: (a) reading the problem, (b) paraphrasing the problem, (c) visualizing the problem on paper or mentally, (d) hypothesize, or plan how to solve the problem, (e) estimating the solution, (f) calculating the answer, and (g) check and evaluate the validity of the process. Additionally, cognitive strategy instruction includes metacognitive strategies such as self-monitoring and self-regulated learning. Students who employ self-regulated learning strategies are aware of their own thoughts and behaviors. They plan and set goals for their learning, use appropriate learning strategies, monitor and question their own performance, and reflect on their performance. These are critical thought processes that students need in their application of adaptive reasoning. Within self-regulated learning is self-monitoring which is a strategy where students actively keep a record of their own behaviors and compare them to a targeted behavior.

Peer-Assisted Learning Strategies (PALS) is an instructional method that was developed by researchers at Vanderbilt University. Students using PALS receive instruction on how to effectively become a peer tutor, then use those skills to tutor another student. This instruction includes how to model, explain, gradually release responsibility to the tutee, and provide feedback. Effective feedback should also be provided by the instructor, as it is another instructional strategy to promote the development of adaptive reasoning (Pulls & Burns, 2022).

Schema-based instruction is explicitly taught, and Powell and Fuchs (2018) describe effective schema instruction as:

> Providing explanations in simple, direct language; modeling efficient solution strategies instead of expecting students to discover strategies on their own; ensuring students have the necessary background knowledge and skills to succeed with those strategies; gradually fading support; providing multiple practice opportunities; and incorporating systematic cumulative review. (p. 6)

Concrete-representational-abstract is a common mathematics instructional method where concepts are first introduced with physical manipulatives, then representations, and are finally displayed abstractly using the numerical model. Think-alouds are another common instructional strategy where teachers model their thinking out loud as they solve a problem. Students then use these models to help them independently solve problems.

Creative Problem-Solving Model

The Creative Problem-Solving (CPS) model of learning is designed to strategically use divergent and convergent thinking processes to find a solution to a problem. This model was developed in the 1940s by Alex Osborn, who was also the person credited for popularizing the term "brainstorming." When using the CPS model, students are given the opportunity to solve problems by identifying challenges, creating ideas, and implementing innovative solutions (Muin et al., 2018). These opportunities are given over long periods of time, where the teacher plays the role of facilitator and motivator, and students are given the chance to solve problems in many different ways. After researching the effect of the CPS model in the mathematics classroom, Muin et al. (2018) concluded that "the creative problem-solving learning model can be used to improve students' mathematical adaptive reasoning abilities, so it can be used as an alternative learning model that can be used by teachers in mathematics learning" (p.6). Currently, there are many different variations of the CPS process which revolve around four main stages. In the first stage, the problem is identified in the form of a question and information is gathered to help clarify the problem. Next, divergent thinking skills are used to brainstorm several different ideas that could potentially address the problem. These ideas are then evaluated to determine the best one. Finally, the accepted idea will be used to formulate a plan to find a solution to the initial problem.

Problem-Based Learning Model

The Problem-Based Learning (PBL) model is designed to engage students in meaningful real-life situations where collaboration is used to help collaborative group develop their own solutions to problems. This process was popularized at McMaster University in Ontario, Canada as an instructional method for medical students. Darwani et al. (2020) were able to show that "learning with the PBL model can foster the ability of adaptive reasoning" in mathematics instruction (p. 4). PBL units can last weeks or even months, but this process should initially be taught with a simple problem and direct instruction on how to navigate that problem. This model follows a similar course as the CPS model where first a problem is presented, and information is gathered. Then, brainstorming occurs to think of multiple potential solutions to the problem. Those potential solutions are discussed, and a final solution is reported on. Throughout this process, the teacher acts as a facilitator to groups of students who actively construct their own knowledge. Key differences between the PBL and CPS model include the PBL model's emphasis on collaboration and the reporting of solutions.

Conclusion

All educators of young students of mathematics should understand the importance of the development of adaptive reasoning in their students and be equipped with a variety of instructional strategies to promote these important skills. The strategies discussed in this paper are not new pedagogical revelations, but each can be incorporated and adapted in the mathematics classroom to enhance students' abilities to discover and justify solutions. Educators who decide to use any of these strategies also need to make sure to fully develop their own understanding of them, and how to successfully implement them in their own classrooms. The development of adaptive reasoning in young students of mathematics will create mathematically proficient students who experience continued engagement and success in the classroom.

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