

What is Error Analysis, and How Can It Be Used in a Mathematics Classroom?

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Abstract: Everyone makes mistakes, but the important part of mistake making is learning and growing after having made the mistake. Mistakes can reveal a lot about the thinking process, and this is valuable information for educators. Instead of treating mistakes as negative events in a student's progress, they can instead be thought of as helpful and critical tools in the learning process. The important question to answer is how teachers can use student errors to successfully create learning opportunities for their students. Teachers can use error analysis in several ways, to diagnose misunderstandings, initiate discussions, and as a starting point for revealing and clearing up misconceptions. Error analysis can create unique learning opportunities for students and should be utilized by teachers.

Learning from Mistakes

I once observed a 7th grade mathematics classroom with varying levels of student achievement. The class started out with a review of the previous night's homework. In other classrooms I had observed, students would typically ask few to no questions at all about problems they missed. This classroom was different, however; here the students were raising their hands left and right to volunteer their mistakes! First, the teacher would ask the student if they knew what they did wrong. If the student was unsure, the teacher would invite the student up to the board to show the work they had. Once the problem was up on the board, the teacher would invite the other students to analyze the error they had made. Other students would then raise their hands to explain what they thought the student had done wrong. The teacher would facilitate the discussion in the class and allow students to reach their own conclusions about where the mistake had been made. The teacher would also provide prompts to students if the discussion stalled, such as "What would happen if the student had done this...?" I was amazed at how, given the right setting, students were able to brush off the negative stigma of getting a question wrong so that they could learn from their mistakes. At this moment I knew I had to learn more about error analysis so I could incorporate it into my own classroom.

Defining Error Analysis

Radatz (1979) was one of the first researchers to look at error analysis in the mathematics classroom. Radatz defined error analysis as specifically looking at the arithmetical errors a student makes and trying to analyze what went wrong with the student's information processing. Radatz looked at error analysis purely from a diagnostic perspective; that is, looking at student errors as a teacher and figuring out what they revealed about where students need the most help. While Radatz was the first to formally describe doing this in his research, this concept is nothing new to any teacher, as many use formative assessments to take note of where students are

struggling and how to adjust instruction based on the errors made in the student work. If teachers do not pay attention to student mistakes and what they mean, those teachers will have a harder time diagnosing learning difficulties in their students.

Analyzing these errors gives the teacher an area of focus of where their students are struggling. For example, the patterns of errors that a teacher may notice include errors in understanding spatial relationships, errors due to language difficulties, and errors due to the misapplication of rules. Finding out which of these (or other) categories a student's error falls under makes it easier for the teacher to come back to an individual student and help them understand where they went wrong in a productive way.

While Radatz (and many teachers) focused solely on the diagnostic power of error analysis, error analysis takes on multiple forms. This article will discuss the importance of and ways to use each form, in the service of empowering students to use discourse in the mathematics classroom in order to master mathematical concepts.

The Springboards of Discourse

Many teachers already know the benefit of looking at student errors without labeling it as "error analysis." Once we focus in on error analysis as a specific concept and practice, however, we can look deeper into how it can be used to further mathematical learning. Borasi (1987) suggests not only that student errors can be used for a diagnostic purpose, but that they can also be used by the students to create discussion. In Borasi's research, students were expected to discuss different types of errors with one another in class instead of just solely correcting their errors and keeping their reasoning to themselves. This in turn created vibrant discourse among students in which the teacher's role became that of a facilitator as the students worked together through what went wrong in one another's work. Borasi explained that teachers should "make use of the potential mathematical errors, both as springboards for problem solving and problem posing and as a grist for critical thinking on the nature of mathematics itself" (p. 2). Errors spark a cognitive dissonance in the mind, and individuals are motivated to relieve the discomfort of that dissonance by fixing the error. Students are thus naturally motivated to correct incorrect work once they are able to move past the negative stigma surrounding errors in the classroom. This type of discourse surrounding errors precisely matches the atmosphere I saw in my initial observation.

In another study, Borasi (1994) set up a quasi-experiment in which students partook in a math class that was treated like a humanities class. At the conclusion of the activities, students were asked to remark on how they felt the learning of the material was affected by error analysis. Borasi found that students had gained a new appreciation of the field of mathematics, and many commented that they had never known that mathematics could be treated like humanities classes, with discussions flowing back and forth. Borasi also found that students were able to grasp some significant mathematical concepts through the use of error analysis. They were able to see their own mistakes and attach meaning to those mistakes which was then readily remembered. Students that discuss what they are thinking through discourse

are able to practice articulating their ideas and to think about whether their reasoning makes sense, as well as to challenge and support one another in doing so. The climate in which this educational discourse takes place is much different than the traditional mathematics classrooms that have students learn procedures, take notes, and practice problems. This type of discourse requires a rethinking of what a mathematics class can be.

Lerman (2000) argues that this cultural change in the math classroom is already happening. He explains that there has been a shift in mathematics education from the transmission of mathematical knowledge to a cognitive focus on student thinking, which is ultimately good news for proponents of error analysis. In part this shift has come about as a response to an ever-changing global market in which U.S. math scores have slipped behind other world powers, which has led to the adoption of a more cognitive approach, following the lead of those other countries. As a result, mathematics classrooms are becoming more centered on how students make math meaningful to themselves, rather than on just the transfer of knowledge through the memorization of equations and rules.

Similarly, Eggleton and Moldavan (2001) specifically examine how error analysis can make discourse in the mathematics classroom meaningful to students. They found that students who work through their own errors were better at recognizing misconceptions in their work later in the year. When students are pre-taught misconceptions, they will not always remember to avoid the same mistakes when taking an assessment. However, these researchers found that when students look at their own mistakes and misconceptions, they attach meaning to that learning because they are learning from their own errors. Not surprisingly, students who worked with analyzing their own mistakes performed better on a follow-up assessment than those students who were warned about misconceptions independent of error analysis.

Eliciting Errors

In order to arrive at the kinds of learning-nurturing discourse described above, students need to make meaningful errors that make visible their misconceptions or misapplications of concepts. To do this, teachers can use a technique developed within the framework of error analysis by creating error-eliciting questions. Lim's article "Error-Eliciting Problems: Fostering Understanding and Thinking" (2014), focuses on creating discussions from problems that are designed specifically to challenge students and to encourage them to make errors. The article explains that showing students common mistakes through their own experience can be a great start for students to understand mathematical concepts at a deeper level. Lim defines an error-eliciting question as a "mathematical task[]... designed specifically to bring forth among students common mistakes pertaining to a particular mathematical concept" (p. 107). Lim described three different types of tasks that can elicit an error. These include tasks that elicit a misconception, tasks that elicit a misapplication, and tasks that elicit an overgeneralization. All three types of tasks are specifically designed to elicit errors from students that show their misunderstandings, and all three of these tasks can be used to create discussions centered on the errors that students make. When a teacher tries to teach against a misconception in a conventional way, such

as by describing the misconception to their students, the student may not develop a meaningful understanding of that misconception. Lim's argument is that the error-eliciting questions can show the student their misconception firsthand and allow them to address the misconception in class before any assessment will take place.

There is now a general three tiered framework in which teachers can use error analysis in their classrooms. The three tiers include the use of error analysis for diagnostic purposes and for the fostering of discussion, and the use of intentional error eliciting to stimulate discussion and learning. Teachers should use the diagnostic side of error analysis to check student progress and remediate any lingering misconceptions. They can also use the diagnostic side of error analysis to evaluate the effectiveness of their lessons and where some conceptual holes may lie. Teachers should use the discussion side of error analysis to create rich discourse in their classroom, so that students are able to address their mistakes explicitly and turn the negative experience of making an error into a positive learning opportunity for themselves and for other students. To create these discussions a teacher may turn to the use of error-eliciting questions, which offer a starting point from which to talk about common misconceptions students have. These three methods make up the totality of what error analysis can look like in a mathematics classroom.

The Importance of Error Analysis

Teachers have a limited amount of time to make their instruction the most effective for their students. Error analysis deserves a chunk of time in the mathematics classroom because of the important discourse it will bring about. The research on error analysis does not lay out one correct way of performing it, but allows teachers flexibility in how they utilize this instructional device. Teachers can therefore tailor their error analysis to fit the diverse needs of their own students.

In Borasi (1996) states that "it is necessary that a compatible learning environment be established in the classroom. Among other things, this may require a change in emphasis from "product to process" (p. 7). This means that for error analysis to occur there must be an attitude of acceptance towards student errors and students must not be afraid to share their mistakes with their peers and their teacher. While this change in student attitudes towards errors may take some time to establish, the payoff will result in deeper conceptual understanding for students and unique learning opportunities that they would not have if they did not analyze their mistakes.

Supporting this attitude toward making mistakes, Kapur (2014), examined whether students would learn mathematical concepts better if they were first taught how to perform the operation or if they were first allowed to try to problem solve without any formal guidance. Kapur found that those who were allowed to problem solve first had a better conceptual understanding of the mathematics, and he also found those students learned from their errors in problem solving. The implications of this study again point towards "productive struggle" as being important for students as they learn mathematical concepts. Teachers, therefore, need to start letting their students explore problems in their own time, and let the process of learning come from a place of exploration and trial-and-error instead of focusing only on getting the correct answer.

Conclusion

The usefulness of error analysis cannot be understated. The method allows students to gain a deeper understanding of concepts and leads to productive discourse. The research completed thus far has not given insight into one “correct” way of performing error analysis, therefore teachers have many options when it comes to bringing error analysis into their classrooms. Teachers can use error analysis as an important tool in their toolbox for helping students learn effectively. Error analysis can lead to a rich interaction among students that helps foster perseverance and offers them opportunities to eliminate misconceptions in a meaningful way.

References

- Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*. Norwood, NJ: Ablex.
- Borasi, R. (1994). Capitalizing on errors as “springboards for inquiry”: A teaching experiment. *Journal for Research in Mathematics Education*, 25(2), 166-208. doi:10.2307/749507
- Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the Learning of Mathematics*, 7(3), 2-8.
- Eggleton, P. J., & Moldavan, C. C. (2001). The value of mistakes. *Mathematics Teaching in the Middle School*, 7(1), 42-47.
- Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, 38(5), 1008-1022. doi:10.1111/cogs.12107
- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44). Westport, CT: Ablex.
- Lim, K. H. (2014). Error-eliciting problems: Fostering understanding and thinking. *Mathematics Teaching in the Middle School*, 20(2), 106-114. doi:10.5951/mathteacmidscho.20.2.0106.
- Radatz, H. (1979). Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3), 163-172. doi:10.2307/748804.



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