Striving for a Conceptual Understanding of Mathematics for All Students

Jessica M. Kuohn

Abstract: The process of learning to learn mathematics starts with an educational approach of allowing the students to discover mathematical concepts. This discovery enforces conceptual understanding in mathematics and eliminates the need for rote memorization of procedures and formulas. For this teaching style to be successful, educators must be willing give students control of their own learning. By allowing students to process mathematical concepts individually, teachers help them make real-life connections and build their knowledge of mathematics collectively and across all grades levels.

Introduction

We regularly hear educators, students and parents say things like, "he was never a math person," or "math was never really my strongest subject." Such beliefs lead individuals to think that they are incapable of being effective mathematical problem solvers at any level. Why do so many people believe that we are incapable of mathematical problem solving? How can we as educators assist students to become problem solvers and eliminate the preconceived notion that not all individuals are mathematical thinkers? This research is based upon the study of successful mathematics teaching as well as of the theories of mathematics; its goal is to provide every student an equal opportunity at accomplishing problem-solving mastery.

Why is Conceptual Understanding in Mathematics Important?

Individuals often believe that they are incapable of mathematical thinking and problem solving, even though this is rarely if ever true. All students (at any level) are capable of becoming great problem solvers, but misconceptions or gaps in one's mathematical education can leave them with "holes" in the web of mathematical knowledge that everyone must acquire in order to successfully proceed in their mathematical education.

According to Dreyfus and Eisenberg (1996), all students possess an equal ability to learn mathematics, and this can be nurtured by teaching them mathematical problem solving skills. In order to obtain these skills, knowledge needs to be created by and within the individual to ensure learning and understanding. Therefore, students should be prompted to construct their own learning of mathematics. When educators do not allow students to do so, they are hindering individuals' learning processes. An educator's role in the classroom is to be a resource, and not to "pour" information into students.

One example can be seen in figure 1, which pertains to students' misconceptions about solving two-step equations. As educators, this is one reason why we need to be very familiar with content we teach--because this familiarity allows for the quick identification of misconceptions so that we can then lead students to the discovery of why their answers are incorrect. Students' mastery and deep understanding of mathematics can essentially eliminate misunderstandings such as the failure to notice that an equation needs to be balanced. In order to do so in figure 1, we must subtract five from each side, otherwise the equation will not remain equal.

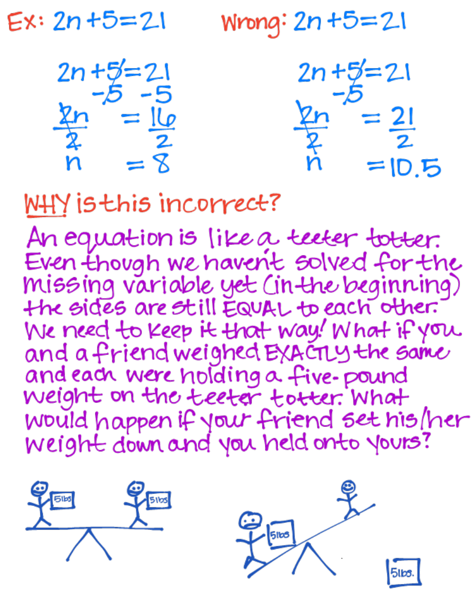

Figure 1: Example of a student's misconception about how to properly solve for a two-step equation, along with an explanation of why it is a misconception.

Classroom Examples

To further explicate misconceptions that are common when teachers rely on teaching by rote memorization, I have included several examples from the classroom that may challenge students and may lead individual students to believe that they are incapable of mathematical thinking. First, we can look at an example of multiplying and dividing fractions. If a student lacks a basic understanding of the relationship between multiplication and division of fractions, this will cause a barrier in algebra when students are expected to multiply and divide to simplify given expressions. Teaching sixth, seventh and eighth grade for the past five years, I have noticed students struggling with remembering how to multiply and divide fractions. Students have memorized the fact that they need to flip one fraction, but they struggle with remembering which operation needs a fraction flipped to the reciprocal, as well as which fraction within the given problem needs to be flipped (the first or the second). I often hear, "keep it, change it, flip it," which is a phrase taught to students to help them memorize the way to solve division problems without actually understanding the reasoning for applying these methods (see figure 2). It usually takes some time at the beginning of the year to address this issue with students. I allow them to engage in discussion and to figure out that division was the operation for which applying this memorized slogan yielded the correct answer. I then present students with a problem where numerators and denominators have common factors, and ask students to solve the problem using that method. Once students have arrived at an answer and have come to a consensus with their classmates, I ask them to solve the problem straight across numerators and denominators, as if they were completing a multiplication problem (see figure 3). Students are often baffled, and instantly believe that their previous teachers have been making them do unnecessary work for years, they question whether the "keep it, change it, flip it" rule in fact, ever needed to be applied.

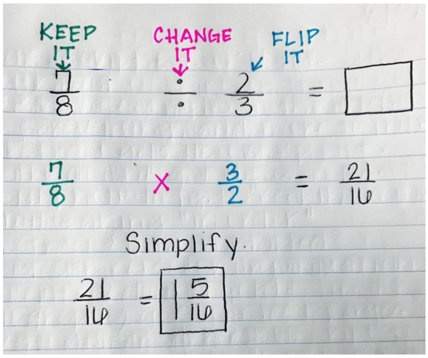

Figure 2: The "Keep It, Change It, Flip It" method often taught in middle grades.

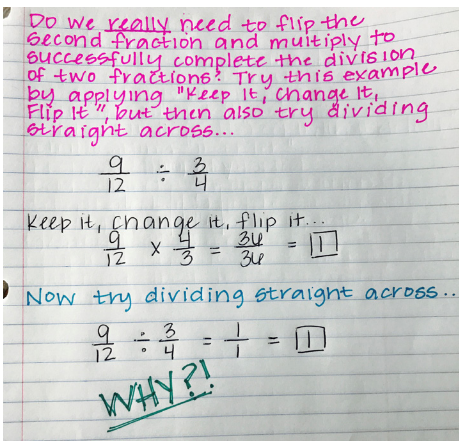

Figure 3: An example of a division problem where flipping the second fraction is not necessary to receive an answer without complex fractions.

To elaborate on this process and relate this example to my research, I allow students to discover that the process of flipping the second fraction in a division problem will yield the same answer as completing the problem without flipping the second fraction to the multiplicative inverse. This process is a simple way to eliminate complex fractions from your final answer. Once students are able to understand why they are doing what they are doing, they will no longer need to memorize an unnecessary "rule" to remind them how to complete a problem. For this reason as educators, we need to strive for the discovery of mathematics for all students, and move away from teaching mathematics through memorization.

A final example often seen in the classroom is students' understanding of the application of the property of exponents (see figure 4). When I began teaching, I used to teach the six properties of exponents through memorization. We would record all properties in books and fill the books with colors in order to be "engaging." Unfortunately, this method did not help students understand why the rules work, and therefore students easily forgot all properties of exponents over time. In figure 4, we can see the discovery process of the "quotient of powers" exponent property. We can focus on the rule and rote memorization, but if we eliminate the memorization and focus instead on understanding we can eliminate the need to memorize formulas completely.

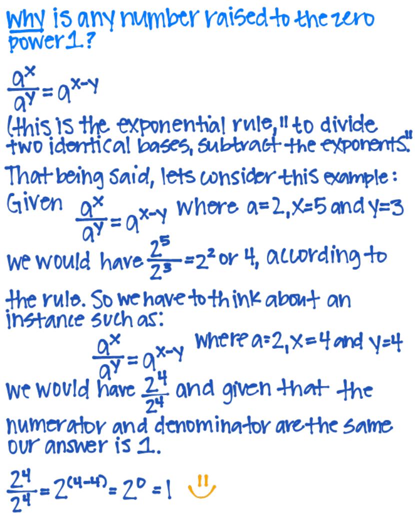

Figure 4: Example of discovery of why exponent properties work.

Why is Discovery Important?

Greer (1992) believes that it is easy for educators to skip the teachings of intermediate representations and move onto the teachings of expressions and surface clues (particularly in the case of multiplication and division), therefore eliminating the probability of conceptual connections being made. These misconceptions and gaps in mathematical education will then carry into students' future mathematical education. For example, if there are underlying deficiencies in the early stages of multiplication and division, then there will most likely be difficultly in conceptual understanding of complex mathematics involving multiplication and division as well.

The Educator's Role

There is a difference between the "doing" and the "understanding" of mathematics. Hiebert and Lefevre (1986) argued that conceptual understanding of mathematics is defined as being rich in relationships. It is a method of connecting the prerequisite knowledge with the current knowledge being learned, and therefore creating a web of mathematical knowledge. Procedural knowledge is composed of formal language and algorithms. Procedures can be learned by rote memorization, yet the understanding of procedures can also be violated by rote learning. Educators must do their best to understand where conceptual knowledge ends and where procedural knowledge begins. In addition, we need to provide students with an education that allows them to tie conceptual and procedural knowledge together as often as possible. By recognizing these methods of teaching, we can once again eliminate the "need" for rote memorization.

Educators can avoid a reliance on student memorization by paying close attention to their instructional methods. Jaworski (2005) therefore explained the importance of educators' knowledge of the curriculum and pedagogy of mathematics in order to understand how to incorporate mathematical activities into the classroom, as well as the necessity of having background knowledge about each individual student in order to be successful. Achieving these goals as an educator is often difficult because of the complexity of education. Each child's experience in education should be student-driven and not teacher-focused.

According to Mayer and Hegarty (1996), research shows that students perform well on state tests that involve basic arithmetic computation, but tend to perform poorly on tests with higher-level skills that involve things like mathematical problem solving. Students are often able to solve mathematical computations, but they cannot apply the same procedures to multi-step word problems. This is one important reason why there needs to be a shift in the curriculum and the way that mathematics is taught. The focus should be on conceptual understanding and making connections to the real world. As educators, we need to recognize this and to reevaluate our teaching methods. We also need to consider how problem solvers solve problems. As stated by Saxe, Dawson, Fall, and Howard (1996), "A fundamental assumption that dominates today's discussions of the psychological nature of mathematical thinking is that it is a construction of the human mind" (p. 120). Mathematical concepts are not created by an individual's environment or through language, but rather are created by individuals based on the relevance of situations within their life. Problem solving happens when a problem solver understands the process of how to arrive at answers, and are less concerned with the answers themselves. And the individual construction of mathematical understanding means that we cannot treat every student the same; we must consider their backgrounds and personal experiences in which mathematics can be related.

Emerging Themes Within the Research

The first emerging theme within my research is the shifting process and approaches toward teaching mathematics. Schoenfeld (1992) and Fuson (1992) have both claimed that educators should be focused on new knowledge about the thinking of students as well as have new goals in the processes of education to assist students in their learning. When we use the term "problem," we should focus on the inquiry-based thinking in mathematics and not routine procedures with rote memorization. Students do not develop problem solving strategies by being "taught" and then completing repetitive problems. Yet this was how mathematics was often taught in the past, which regularly resulted in gaps of understanding.

A second emerging theme is the importance of teacher knowledge. According to Fennema and Franke (1992), teachers' understanding of content is one of the most important factors of teaching. It is imperative that we are able to understand misconceptions of students in order to "fix" problems and fill in the holes where given content is not understood. We can give a student as many problems to solve as we would like, but if they do not understand why they are doing what they are doing, there is no point to the repetition. Jaworski (2005) believed that it is important to get learners to learn based on discovery, but that educators need to guide students in the right direction so that they learn what they need to learn. Without our guidance in mathematical discovery, students may stray away from the curriculum in which they need to learn.

Readings on modern teaching strategies also stress the importance of educators' content knowledge. If an educator has not mastered and understood a given mathematical concept, they will be unable to recognize the misconceptions of their students, and will most likely fail to identify a way to guide students to the approach through discovery. In addition, teachers without content mastery are typically incapable of providing students with the proper resources to create an engaging environment.

Another emerging theme can be found in almost every example of recent research: every individual is capable of learning mathematics. Dreyfus et al (1996) believe as educators, we should never assume that some people cannot grasp mathematical problem solving skills. Thinking patterns in mathematics can always be learned.

A further theme with the research relates to the cultural background of individuals. When we think about making learning engaging, it doesn't necessarily mean we need to utilize puzzles, coloring, and activities to make the learning fun. A truly engaging environment is one in which students are interested in the learning, and can relate their mathematical discoveries to real-life situations. Given that everyone has a different background, their methods of learning and engagement in the content should be unique to their own personal experiences, and a variety of experiences should be offered.

The last theme is the avoidance of rote memorization. One of the biggest problems in the classroom is teachers treating mathematical problem solving as a process of rote memorization instead of focusing on actual problem solving and conceptual mastery. It is important for children to learn numbers in their own cultural experience. If students lack this conceptual understanding, the mathematics that is taught in younger grades can directly affect the comprehension of mathematics in later grades. For example, Schoenfeld (1992) tells us when we use the term "problem", we should focus on the inquiry-based thinking of mathematics and not routine procedures with rote memorization.

Conclusion

Educators need to continue their education so that they can provide the best learning experiences for all students. This doesn't necessarily mean that all educators need to attend continuing education classes, but rather that they should adopt new methods of teaching, focused on allowing students to learn how to learn by themselves. A focus on mathematical concepts will allow students to have a conceptual understanding of mathematics, and therefore can eliminate the need of rote memorization.

References

Dreyfus, T., & Eisenberg, T. (1996). On different facets of mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 253-284). Mahwah, NJ: Lawrence Erlbaum Associates.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Fuson, K. C. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 243-275). New York: Macmillan.

Greer, B. (1992). Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of research on mathematics and learning* (pp. 276-295). New York: Macmillan.

Hiebert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge in mathematics: An introductory analysis*. Hillsdale, NJ: Lawrence Erlbaun Associates.

Jaworski, B. (2005). Development of the mathematics teacher educator and its relation to teaching development. In T. Wood (Ed.), *International handbook of mathematics teacher education* (Vol. 4, pp. 1-27). Rotterdam, The Netherlands: Sense Publishers.

Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematical problems In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 29-53). Mahwah, NJ: Lawrence Erlbaum Associates.

Saxe, G. B., Dawson, V., Fall, R., & Howard, S. (1996). Culture and children's mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 119-144). Mahwah, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

About the Author

Jessica M. Kuohn earned a Bachelor of Education in Middle Childhood Education with concentrations in mathematics and science in 2012 and a Master of Education with a focus in mathematics in 2017. She has been an educator of mathematics for five years and strives for mastery in mathematics for all students.