

Changing the Way That Math is Taught Conceptual Versus Procedural Knowledge

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Abstract: The purpose of this article is to present teachers with information and data regarding the relationship that exists between procedural and conceptual knowledge while attempting to validate the notion that the development of conceptual knowledge should be at the forefront of student learning in the mathematics classroom. The article first delves into the definitions and overall significance that each of these modalities have in the mathematics classroom and after doing so, the article examines the theoretical ideologies that encompass them. The article concludes with a look into how the two knowledge bases are related, as well as empirical evidence regarding which is better for effective classroom instruction.

Keywords: conceptual, procedural, instruction, knowledge, math, teaching

Introduction

Think of your earliest mathematics educational experience. For me, as well many others, it is probably completing routine, rote math problems to the point where you really do not know what you are doing, or how it is related to anything mathematical. You are simply following a predetermined set of rules that the teacher has given to you to solve each problem. There is no way of knowing if you are making mistakes, or if there is more to the problem than simply computing random digits using seemingly arbitrary information that may or may not apply to anything in the real-world.

For context purposes, think of how you were taught to divide fractions. Across the field, nearly all educators teach some variation of keeping the first fraction the same and multiplying by the inverse of the second term in order to get the solution. This is how it has always been done, and it certainly does its job in getting the students the correct answer, however, do students really understand what they are doing, or are they simply following a prescribed plan that the teacher taught them? Is it possible there is more to learning math than simply memorizing formulas, procedures, and applying random digits?

Now, picture a classroom where students are again working with division of fractions, but this time they are doing so with fraction strips and other manipulatives that allow the students to conceptualize this difficult topic in a meaningful manner. Instead of simply being told how to divide fractions, students are instead working together, discussing ideas, experimenting and exploring complex ideas to develop their own procedures for dividing fractions. There is far less explicit instruction by the teacher and the students are taking an active role in their own learning as well as engaging the materials in a meaningful manner.

The big questions educators have to ask concerning these two very distinct styles is: what is known about the effects of conceptual and procedural understanding in relation to the ability for students to retain mathematical ideas and concepts? Are the results for students different for type of instructional method, or is this simply a case of two ways to achieve the same result? How are the two modalities related? These are the pressing questions every new and long-standing mathematics instructor should be asking themselves. The answers to these questions directly impact how educators should deliver mathematics instruction to their students as well as how the students will engage in learning.

This article will focus on exploring the relationships that exists between conceptual and procedural knowledge, and attempt to establish the importance of a mathematics classroom that is based around the development of conceptual knowledge. There is no doubt that each type of knowledge is important and has its place in the mathematics classroom, however, it appears that conceptual knowledge should be at the forefront of all classroom learning.

The National Council of Teachers of Mathematics or NCTM (2014) has articulated this emphasis on conceptual knowledge by calling for decreased attention to learning procedures without any connection to meaning, understanding, or the applications that require these procedures. Are they correct in doing so, or is this an overly-complicated educational trend with no real basis?

Definitions

According to Hiebert (1986), conceptual knowledge is characterized as “knowledge that is rich in relationships. It is a connected web of knowledge in which the linking relationships pervade the individual facts so that all pieces of information are linked. The development of this conceptual knowledge can only be done so by the construction of relationships between pieces of information” (p. 3). Hiebert (1986) defined procedural knowledge as two distinct parts. “The first part is the formal language of mathematics, or the symbol representation system. It includes a familiarity with the symbols used to represent mathematical ideas and an awareness of the syntactic rules for writing

symbols in an acceptable form. The second portion of procedural knowledge consists of rules, algorithms, or procedures used to solve mathematical tasks. These are better described as step-by-step instructions for how to complete a task” (p. 6).

Theoretical Constructs

If anyone looked into a math classroom across the United States right now they would see variations of the following three theoretical constructs regarding conceptual and procedural understanding in math. The first theory, which best characterizes a procedural-based classroom, is known as drill theory. The basic traditions of this theory state that children learn best when imitating the skills and knowledge of adults, understanding is not necessary for the formation of relationships amongst ideas, and the most efficient manner in which relationships form is through direct instruction and drill (Baroody & Dowker, 2003). This parallels the first classroom that was described at the outset of this paper and is the exact environment that the NCTM is trying to eliminate in the math classroom.

The opposing, reactionary theory to drill theory is the incidental-learning theory, which according to Baroody and Dowker (2003) states that “children should be free to explore the world around them, notice regularities, and actively construct their own understanding and procedures” (p. 7). This is the theory that children should be learning as a result of their natural curiosity in mathematics. Although this may seem great in theory, it is impractical due to its time-consuming nature, its fragmented, unfocused nature, and most teachers and schools lack the ability to implement it effectively. This theory might work under the controlled circumstances with the right group of students, however, because of the deficiencies listed above, it is simply not practical.

The middle ground between these two theories is the meaning theory. This theory approaches learning from the perspective that students, at first, should engage in self-invented reasoning strategies prior to formal instruction as they provide the foundation for more developed knowledge and mastery. Instead of solely relying on drilling, the meaning theory instead uses it in a complementary role to increase permanence of recall. In further opposition to drill theory, the meaning theory allows time for students to construct an understanding of ideas and relationships that exist in math, instead of simply memorizing facts that mean nothing to them (Baroody & Dowker, 2003). This theory would match up nicely with the second classroom that was described at the outset of the paper and is quite similar to what the NCTM is promoting, which is a conceptually based classroom.

As a mathematics instructor, one of these theories should stick out as the best, or look familiar. Proponents of the drill theory believe that math instruction should focus on promoting mastery of basic skills, while putting little emphasis on understanding of concepts. Advocates of meaning theory, on the other hand, recommend the use of instruction and drill to promote skill mastery, but recognize the value of building students’ experiences in discovery and conceptual understanding (Brownell, 1935). The incidental-learning theory is one that also focuses on the development of conceptual knowledge, but in a more informal setting where the students may or may not even know that they are learning.

Relationship between Conceptual and Procedural Knowledge

There have been countless studies conducted in order to determine the best way in which students learn mathematics and these studies almost always boil down to one simple idea: the relationship between procedural and conceptual knowledge. Byrnes and Wasik (1991) describes one view which is known as simultaneous activation, or the idea that students’ errors in math arise from the fact that mathematical symbols are meaningless to them. The argument for this stems from the idea that symbols are meaningless because computations are typically learned in a rote fashion or procedural knowledge. According to this view, errors are due to a low conceptual knowledge base.

The contrasting, dynamic interaction, view presented by Byrnes and Wasik (1991) states that procedures are developed due to a rich conceptual knowledge that allows the students to transfer procedures and ideas to new contexts. This view postulates that conceptual knowledge facilitates procedural knowledge and would effectively eliminate any reason for teachers to instruct their students in a rote, conceptually lacking manner. One study conducted independently by Byrnes (1992) directly supports this view and states that the students that had the “most conceptual understanding before treatment [instruction] came out of the study gaining the most procedural knowledge” (p. 237). This directly implies that it’s in the students’ best interest to have a solid base of conceptual knowledge in order to also be procedurally-sound.

Rittle-Johnson and Alibali (1999) also investigated this relationship by looking at the impact that conceptual understanding had on procedural understanding and vice versa. The main point is that teachers should see where the most gains are being made by their students as this would allow teachers to streamline and improve their teaching practices. Rittle-Johnson and Alibali (1999) found that there are several ways in which the development of conceptual knowledge impacts procedural understanding, with the first being that the greater the conceptual knowledge, the greater the procedural skill, which directly supports the earlier statement by Byrnes (1992). The second idea is

that conceptual knowledge naturally precedes procedural understanding in many cases. For example, some preschoolers already understand principles of counting when they first learn to count (Gelman & Meck, 1983). Third, the instruction about concepts, as well as procedures can lead to increased procedural skill, and lastly, increased conceptual knowledge can lead to increased procedure generation (Rittle-Johnson & Alibali, 1999).

This relationship does not seem to be unidirectional. In fact, they seem to influence one another. There is even a bidirectional relationship that exists (Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Schneider & Star, 2015). Children that received conceptual instruction not only increased their conceptual understanding, but also generated several correct, flexible problem-solving procedures, whereas children that received procedural instruction adopted correct problem-solving procedures and increased their conceptual understanding. This might seem to suggest that conceptual knowledge is no more important than procedural knowledge, however, the gains made between these two modes were not symmetrical in nature and therefore do not suggest ambivalence. Children in the procedural-instruction group had significantly lower transfer performances, which limits the improvement that students can make conceptually. In contrast, gains in conceptual understanding led to fairly consistent improvements in procedural knowledge. Students that received conceptual instruction were just as likely to learn a correct procedure and were better able to transfer their knowledge (Rittle-Johnson & Alibali, 1999).

Rittle-Johnson, Siegler, and Alibali (2001) confirmed these ideas in their study that investigated 5th and 6th grade students and their development of conceptual understanding and procedural skill under the topic of decimal fractions. What they found was that the neither of these modalities develop in an all-or-none fashion, with acquisition of one type of knowledge always preceding the other. In fact, they found that “the two developed in an iterative, hand-over-hand process” (Rittle-Johnson, Siegler, & Alibali, 2001, p. 360). Similar to Rittle-Johnson and Alibali (1999), this study found that improvements in one often led to improvements in the other. These two studies seem to suggest that true proficiency in a mathematical domain, or topic, requires knowledge of both concepts and procedures.

Together, But Not Equal

A fundamental focus of inquiry based mathematics, or conceptually based instruction, is the idea that students explain their thinking. This may not seem like something out of the ordinary, however, when talking about mathematical ideas, teachers should be pressing students to justify their strategies and thoughts from a mathematical perspective instead of simply having the students describe the steps they took to solve a problem. Kazemi and Stipek (2001) showed this importance by investigating student learning in a high-press environment, or one that pushes for conceptual understanding, and in low-press environment, or one that pushed students to simply explain the steps they took to complete problems. According to the results, superficial, procedural understanding is not getting the job done in the classroom, and there seems to be a consensus that in order to promote the development of students’ mathematical ideas, there needs to be a push for the following ideas: increased mathematical argumentation, mathematical thinking involving understanding relations among strategies, using errors provide opportunities to re-conceptualize a problem, and collaborative work involving reaching a consensus through mathematical argumentation (Kazemi & Stipek, 2001). In layman’s terms, this means that there should be an increased push for conceptually based instruction. In support of this idea, Hallett, Nunes, and Bryant (2010) also found data that supports the notion that conceptual approaches will be more successful than procedural approaches in supporting mathematical learning, which they found by comparing a higher conceptual-lower procedural group with a higher procedural-lower conceptual group. Finally, Hiebert and Wearne (1996) also support this notion in their study, which found that students who demonstrated conceptual understanding are more likely than their peers to invent and modify procedures. The results of these studies, along with the findings of the Rittle-Johnson and Alibali (1999) study, suggest that conceptually centered learning leads to better results in the classroom and that developing understanding early in education leads to great results in the future.

Conclusion

Overall, a clear link between conceptual and procedural knowledge exists in the mathematics classroom with both having their place in student mastery. There is an overall consensus that building relationships between conceptual knowledge and procedural knowledge leads to great improvements in procedural understanding, both from a symbolic and procedure-transferring standpoint, however, procedural understanding does contribute to conceptual knowledge, albeit in a lesser sense, by providing formal language and overall enhancement of the applicability of conceptual knowledge. Despite the fact that these two styles grow together and enhance one another, there is evidence that suggests that this is not an equal trade-off. Conceptual understanding improves procedural knowledge more than procedural understanding improves conceptual knowledge. With that in mind, it is clear that mathematics instructors should be pushing for more conceptually based classrooms and moving away from, but not eliminating, procedure-based learning environments.

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