Fostering Mathematical Creativity

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Abstract: The world needs creative problem solvers, perhaps now more than ever. The mathematics classroom is the perfect place to cultivate such creative thinkers. Mathematics is often considered a rule-based subject that has little room for creativity. This manuscript aims to show that creativity has a place in the mathematics classroom. Teacher choices have a significant impact on whether creativity is fostered or suppressed in the math classroom. Through critical reflection of current teaching methods, teachers can create a classroom environment where creativity flourishes. Methods for doing so are discussed, including teaching for understanding, choosing and creating rich tasks that have a "lower floor and higher ceiling," (Boaler, 2016) allowing ample time for thinking, and telling students to be creative.

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The unprecedented challenges we face today highlight the need for innovative and imaginative thinkers. In this rapidly changing world, the future is uncertain. The World Economic Forum reported in 2016 that by 2020, creativity would be one of the top three skills needed by workers. Creativity ranked 10th in the 2015 version of the same list (Gray, 2016). This leap is likely due to rapid advancements in technology that require creative thinkers to make the most use of those technologies. At the same time, creative thinking scores, as evaluated by the Torrance Tests of Creativity, have declined (Kim, 2011). Methods of math teaching that encourage rote learning and procedural knowledge no longer have the clear value they once had, as suggested in the National Research Council's Adding It Up: Helping Children Learn Mathematics (NRC, 2001). Yet these methods are still used, stifling creativity.

Effective mathematics instruction can help to improve the creative thinking skills necessary to improve society. Choices made by math teachers have a significant impact on whether creativity is fostered or suppressed. These include choices about what types of problems to present, how to present them, and how to respond to alternative ideas and solutions given by learners. Through reflection and critical evaluation of current teaching methods, math teachers can make adjustments that allow for a classroom where creativity flourishes.

Definitions and Conceptions of Creativity

In order to explore mathematical creativity, it is helpful to first consider both historical definitions as well as general conceptions of creativity. Although no universally accepted definition of creativity seems to exist, common elements can be found which help to shed light on the essence of creativity.

Definitions

In The Standard Definition of Creativity, Runco (2012) draws parallels between different historical definitions of creativity. He points out that the use of the word itself has a relatively short history within the research, and that the standard definition involves two main elements: originality and effectiveness (Runco & Jaeger, 2012). Original ideas are not considered creative unless they have some purpose or utility. He points to Royce's (1898) use of the term "valuable inventiveness" and Hutchinson's (1931) description of creativity that includes elements of practicality. Runco also gives credit to Barron (1955) and Stein (1953) for their work on defining creative behavior, both whose definitions contain elements of originality or inventiveness along with elements of practicality or usefulness.

Vygotsky (2004) states that "any human act that gives rise to something new is a creative act, regardless of whether what is created is a physical object or some mental or emotional construct that lives within the person who created it and is known only to him" (p.7). Creative behavior combines past experiences and reworks or adapts that knowledge in order to create something new. This activity is what Vygotsky (2004) identifies as the driving force that makes a human being "oriented toward the future, creating the future, and thus altering his own present" (p. 9). Imagination is defined as the basis of all creative activity, being a component of all types of creation and having importance in all aspects of cultural life.

Csikszentmihaliyi (1997) claims that creativity results from the interaction of a system composed of three elements: "a culture that contains symbolic rules, a person who brings novelty into the symbolic domain, and a field of experts who recognize and validate the innovation" (p. 6). In this view, creativity is not an individual phenomenon that occurs inside a person's head. Creativity takes place through the interaction between a person's thoughts and a sociocultural context.

Guilford (1967) suggests that the creative process is based on a combination of convergent and divergent thinking. Convergent thinking involves aiming for a single, correct solution to a problem. Divergent thinking involves generation of multiple answers to a problem.

Torrance (1974) defines creativity as being comprised of four components: fluency, flexibility, originality, and elaboration. Fluency refers to the "continuity of ideas, flow of associations, and use of basic and universal knowledge" (Leikin, 2013, p. 386). Flexibility involves being able to look at a problem from a variety of perspectives, change approaches, and produce a variety of solutions. Originality, often considered the main component of creativity, refers to the ability to generate novel ideas and products. Elaboration encompasses the ability to "describe, illuminate, and generalize" strategies and ideas (Leiken, 2013).

Conceptions

Robinson discusses the misconception that creativity is only about "special people" and is a fixed trait that you either have or you don't (Azzam, 2009). Vygotsky similarly discusses the "everyday understanding" of creativity, belonging to a few selected extraordinary or gifted individuals. This differs from the scientific definition of creativity, which is present whenever a person "imagines, combines, alters, and creates something new" no matter the scope of the result (p. 10).

Creativity is often misconceived as being relative to only the arts and not to science or mathematics. Yet mathematics was created by human beings. Some of the most famous theorems and elegant proofs were creative in nature. Robinson's claim that, "creativity is really a function of everything we do," validates the role of creativity in all subjects (Azzam, 2009, p. 23).

Another commonly held misconception of creativity is that it is free and unstructured. Creativity, however, cannot occur without an individual taking part in some activity of which they understand the structure and "rules" that already exist. While requiring imagination and inspiration, creativity is a "disciplined process that requires skill, knowledge, and control" (Azzam, 2009, p. 23).

Mathematical Creativity

Like general creativity, no standard definition of mathematical creativity exists. Aldous (2005) identifies three elements of creative problem solving in order to construct a conceptual framework for mathematical creativity. Creativity in solving a challenging problem involves interaction between areas of the brain involved in visual-spatial and linguistic activity, the first element of the framework. The second element involves the interplay between rational conscious activity and experiential, non-conscious activity. She describes the way feeling and intuition lead problem solvers to alternate between conscious and non-conscious activity in order to "evaluate, monitor, and filter a particular solution path" (Aldous, 2005, p. 53). According to Aldous (2005), the role of intuition and feeling in problem solving is backed by findings in neuroscience.

Sriraman (2004) also explored the nature of mathematical creativity by studying professional mathematicians as they solved a problem. He was interested in the Gestalt model of mathematical creativity, the characteristics of the creative process, and the implications for the classroom. The Gestalt model of the creative process involves four stages: preparation, incubation, illumination, and verification (Wallas, 1926, p. 10). In general, Sriraman found that the thinking process of the mathematicians interviewed followed this model. The accounts given described a lengthy amount of time in which the participants spent researching the problem and its context (the preparation phase.) They described social aspects of this phase which involved discussing the problem at hand with other experts. Most mentioned working on more than one problem at a time, using a back-and-forth kind of approach, as well as the types of imagery used when investigating an idea. Participants touched on the incubation/illumination phase in which an idea is left to sit for a time before which some type of "aha" moment occurs, similar to the role of intuition described by Aldous (2005). As a final stage, some sort of formal proof was developed (Sririman, 2004).

Boaler (2016) points out a component of mathematical creativity that is strongly linked to the concept of fluency previously described. She discusses compression and the importance of making connections among ideas:

> When you learn a new area of mathematics that you know nothing about, it takes up a large space in your brain, as you need to think hard about how it works and how the ideas relate to other ideas. But the mathematics you have learned before and know well, such as addition, takes up a small, compact space in your brain. You can use it easily without thinking about it (Boaler, 2016).

Fostering Creativity in the Mathematics Classroom

The definitions of creativity and mathematical creativity have several implications for the ways in which teachers can foster creative thinking in the math classroom. These include teaching conceptually, choosing rich tasks, being thoughtful about the presentation of tasks, and creating an environment in which alternative ideas and solutions are accepted and encouraged.

Teach for Understanding

Mathematics is often viewed, and taught, as a structured, rule-oriented discipline. There are rules to be learned and practiced, with little connection to real life. In this commonly held view, there is no room for creativity. According to Boaler (2016), "when students see math as a series of short questions, they cannot see the role for their own inner growth and learning" (p. 34). Instead, learners should be led to see mathematics as a set of ideas and relationships that make sense and are connected. A radical change in the concepts taught is not necessary, but rather a change in the way those concepts are taught. Learners still need to learn and practice fundamental concepts, but "practice" should be revisiting ideas in different ways. This can help increase the components of fluency and flexibility needed for creative thinking. Students should be asked to convince, reason, and be skeptical, allowing students to make connections between concepts and understand the mathematics involved.

Choose and Create Rich Tasks

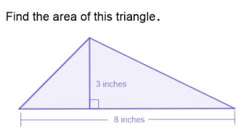
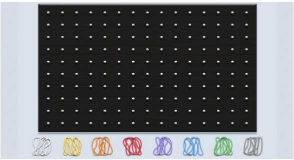
Along with teaching for understanding, creativity and specifically, mathematical creativity can be encouraged by implementing create rich tasks in the lessons design. These tasks include the transformation of traditional problems and the approaches to finding solutions, allowing ample time for creativity to develop, valuing and validating alternative and creative solutions, and asking students to be creative.

Transform Traditional Problems

Boaler (2016) makes several recommendations for the design of tasks that provide opportunities for mathematical creativity for all students. One suggestion is to open up the task so that there are multiple methods or pathways and representations. This can be accomplished by transforming a standard task into an inquiry task, shifting the role of the student from reproducer of a method to an originator of ideas. Many textbook authors in the United States isolate methods in mathematics and reduce them to their simplest form in order to practice. This can lead to boredom in students, corroborating the view of math as a set of disconnected ideas (Boaler, 2016). Teachers can significantly increase the opportunity for creativity with minimal adjustments to "undo" the simplification of traditional problems. Table 1 suggests ways in which problems can be transformed.

Table 1

Transformations of Traditional Problems

Traditional Problem	Alternative
Solve this system of equations: 3x+2y = 6 5x-3y=18	Write a system of equations that has the solution (4, -3). Can you come up with more than one? Can you include a nonlinear equation? OR Here is a system of equations: 3x+2y = 6 5x-3y=18 Can you write a story problem that this system would solve?
Find the area of this triangle: 	Create as many triangles as you can that have an area of 12 units2 on the interactive geoboard. 
Simplify the radical expression: √(112x^5 y^6 z)	Here is a simplified radical expression: 4x^2 y^3 √7xz What might have been the original expression? Can you create more than one?

Pose the Problem First

Instead of introducing a method or algorithm first and then showing where it can be used, pose the problem first (Boaler, 2016). Allow students to grapple with a problem, encouraging creative methods and thinking, and then introduce the method. This gives students a reason to want to learn the formal algorithm. The following table gives some examples of what this might look like in an Algebra 1 classroom.

Table 2

Introducing the Problem First

Problem (introduced first)	Method or Algorithm (taught after students grapple with the problem)
The marketing team for the Toledo Walleye needs your help! They want to know whether to focus their advertising on children or adults. Tickets for children cost $15 and tickets for adults cost $32.00. They know that for the last game, 6,000 people were in attendance and that the total gate revenue was $137,600. How can you use this information to help decide whether more children or adults were in attendance?	Solve a system of linear equations using graphing, substitution, or elimination.

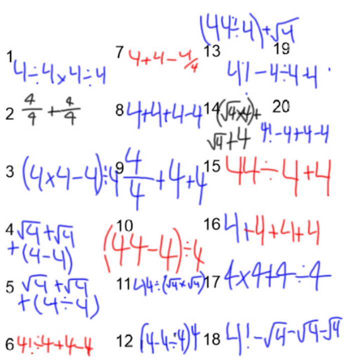
Problem (introduced first)	Method or Algorithm (taught after students grapple with the problem)
A tennis ball is thrown straight up, from 3 m above the ground, with a velocity of 14 m/s. The height of the ball is given by the equation h=3+14t-5t^2 where h = the height of the ball in meters and t = time in seconds. When will the ball hit the ground? Guess and check!	Factoring quadratic equations

Lower Floor, Higher Ceiling

Another suggestion made by Boaler (2016) is to make the task have a "lower floor and higher ceiling" (pp.84-85). The floor describes the entry point to a problem; a lower floor means that the task is accessible to and easily started by all students. The ceiling refers to the task's potential to grow, with a high ceiling indicating that a task increases in complexity, allowing higher achievers to explore the problem in depth without becoming bored (Boaler, 2016, pp. 84, 85).

The "Four 4s" task is an excellent example of a lower floor, higher ceiling task. The problem asks students to make all numbers within a certain range by using only four 4s and any mathematical operation (Figure 1). All students have an entry point to the task, and students often start with four 4s and see what numbers they come up with. The task increases in complexity as certain numbers are more difficult to make than others, leading to solutions involving advanced operations.

Figure 1
Some Solutions to the Four 4s Task.



Allow Ample Time

A theme that emerges in the research about creativity, both general and mathematical, is that learners need ample time to fully think through and understand concepts. Hawkins (1974) points to the crucial importance of the "Messing About" phase where students are given ample time to explore and play with a problem. Similarly, Su (2017) discusses the importance of mathematical play. Teachers frequently refer to a lack of time, pointing to pressures of testing and the need to cover all of the standards. Allowing plenty of time for "Messing About" with ideas can lead to greater and deeper understanding of a concept. This cannot be replaced by merely telling or showing the answer. Slowing down and letting students come to their own understanding is a worthwhile endeavor.

Value Alternative and Creative Solutions

For students to think creatively, they must be assured that the environment is one in which they can try out different ideas and offer creative solutions without a fear of being corrected or shut down. Imagine the effects the teacher's behavior has in the following vignette:

"Who can find the area of this parallelogram? Henry?"

"Well, I just thought that I could cut the triangle off one end and slide it down to the other end and then it is a rectangle that is 8 units by 5 units, so the area is 40 units squared."

"Okay, but we don't need to do any "moving" of pieces since we have the formula, area equals base times height. Isn't that a much easier way of finding it?" The rest of the class nods in agreement.

This brief interaction has the potential to crush Henry's feelings of self-efficacy and self-worth. Consider the alternative scenario:

Who can find the area of this parallelogram? the teacher asks. "Henry?"

"Well, I just thought that I could cut the triangle off one end and slide it down to the other end and then it is a rectangle that is 8 units by 5 units, so the area is 40 units squared."

"Class, what do you think about Henry's idea? Will this always work? How do you know?"

Students conclude that the method will always work.

"Let's call this 'Henry's method.'

In a subsequent class the students are asked to find the area of an isosceles trapezoid. Livia gives a correct answer of 100 square units.

"How did you get your answer?"

"I used Henry's method and moved the triangle on the end down to the other end to create a rectangle!"

For students to think creatively, the classroom should be a place where students feel comfortable trying new ideas and methods, even when they don't work. The reactions and responses of the teacher are crucial in that they can influence a student's willingness to try creative approaches. Mistakes should be valued and used as a means to further discuss the concepts involved along with the validity of a particular strategy.

Tell Students to Be Creative

Sometimes creativity can be encouraged by just telling students to be creative. O'Neal and Runco (2016) described a study in which two groups were asked to devise solutions to a real-world problem. The first group was told to give solutions that were creative, and that no one else would come up with. The second group was simply asked to generate solutions. The first group developed solutions that were significantly more original and creative (Runco, 2016).

Conclusion

In a talk given upon his departure as President of the Mathematical Association of America, Francis Su posed the question: Why do mathematics? He stated that the question was simple yet worthwhile, because how you answer will strongly determine who you think should be doing mathematics, and how you will teach it (Su, 2017). Su's answer is that mathematics is for human flourishing, and includes notions of play, beauty, truth, justice, and love (Su, 2017). Mathematics helps us to make sense of the world around us. It is more than just a set of unconnected rules and procedures. As teachers, we have the ability to influence our students to see mathematics in this light, to encourage creative thinking, and to help transform our students into the innovators of the future.

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